Numerical High Performance Algorithms WS 2018

# Homework 2: Inverse Iteration vs. Rayleigh Quotient Iteration

Implement Inverse Iteration and Rayleigh Quotient Iteration (both based on LU factorization with partial pivoting for solving the linear system) in Octave/Matlab as efficiently as possible.

1. Construct suitable test matrices.
2. Vary the problem size n = 100, 200, 300, … (up to whatever is easily possible)
3. **Inverse Iteration**:
   1. Investigate the influence of the distance between the shift and the eigenvalue computed (quality of the shift as an eigenvalue approximation) on the convergence speed
   2. Investigate the influence of the choice of starting vector on the convergence speed (compare various canonical starting vectors and a random one)
4. **RQI**:
   1. Investigate the influence of the choice of starting vector on the convergence speed (compare various canonical starting vectors and a random one)
5. Compare the two algorithms in terms of overall runtime as well as in terms of numbers of iterations for computing a single eigenpair of a matrix .
   1. Choose an initial shift and use the same random starting vector such that a fair comparison is possible.
   2. Use the same suitable convergence criterion in both cases to ensure that both solutions have the same level of accuracy.
6. What are the inherent disadvantages of RQI? Construct a problem setting, where standard RQI has a clear performance advantage over Inverse Iteration. Describe your problem setting in detail and illustrate the performance comparison experimentally.
7. Consider a variant of RQI, where the shift is not updated in every iteration, but only in every iteration for Compare this variant with inverse iteration and with standard RQI in terms of performance.

Submit your source code and a report (documentation) which summarizes all your experiments (also graphically). Your report has to include the following information:

* How did you construct your test matrices?
* Which convergence criterion did you use?
* Which accuracy was achieved at the time of termination? Measure the accuracy compared to a reference solution (e.g., returned by the eig-function) in the following two metrics:
  + *Relative error in the eigenvalue*: , where λ is the reference eigenvalue and is the eigenvalue approximation computed by your routine.
  + *Norm of the residual vector*: where is the eigenvector approximation computed by your routine.
* Figures with absolute runtimes, numbers of iterations until convergence, average runtime per iteration, convergence history (x-axis: iteration number, y-axis: accuracy metrics *using log-scale*) for all algorithms and all test cases.
* Efficiency values (which percentage of the theoretical peak performance was achieved)
* Interpretation of the observed experimental results.

*Due date* (*submission of report and code in Moodle*): **19.11.2018, 18:00**